MODULAR ATTITUDE CONTROL

OF A LARGE SPACE PLATFORM

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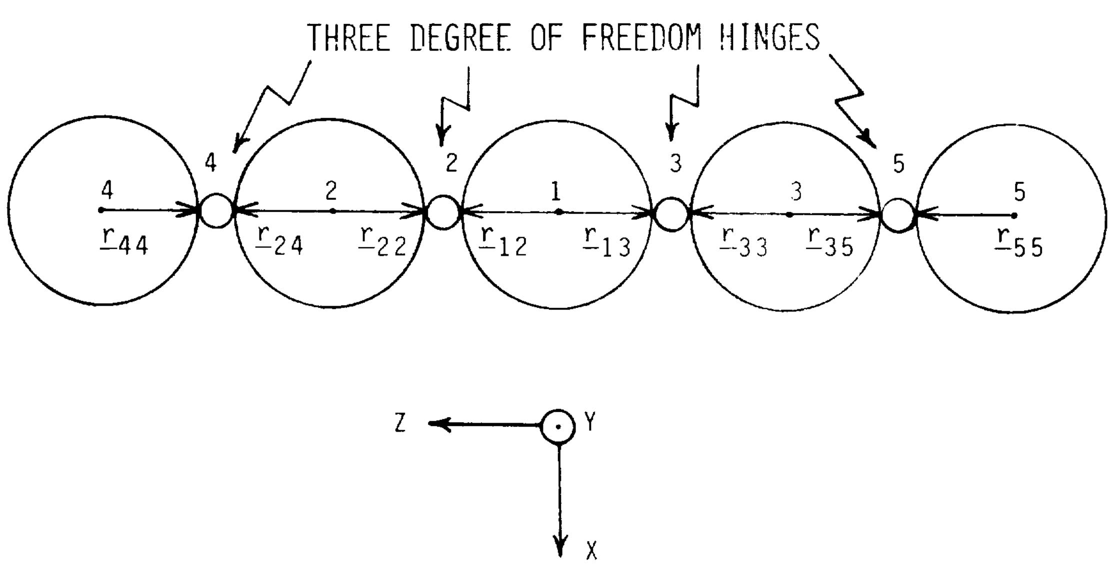
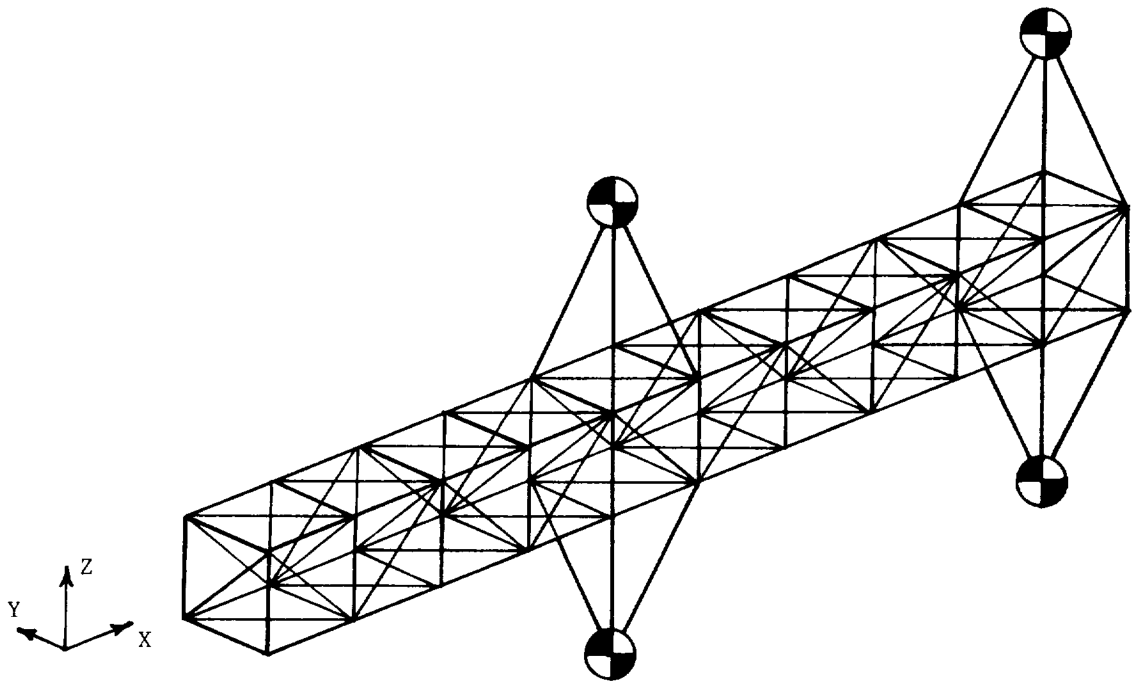
- STATE VARIABLE MODEL
- DECOMPOSED MODEL
- DECOMPOSED PERFORMANCE INDEX
- DECOMPOSED HAMILTONIAN
- COSTATE EQUATIONS
- CONTROL EQUATIONS
- TPBV FORMULATION OF FIRST LEVEL SUBPROBLEMS
- SECOND LEVEL COORDINATION SUBPROBLEMS
- SUBPROBLEM HIERARCHY

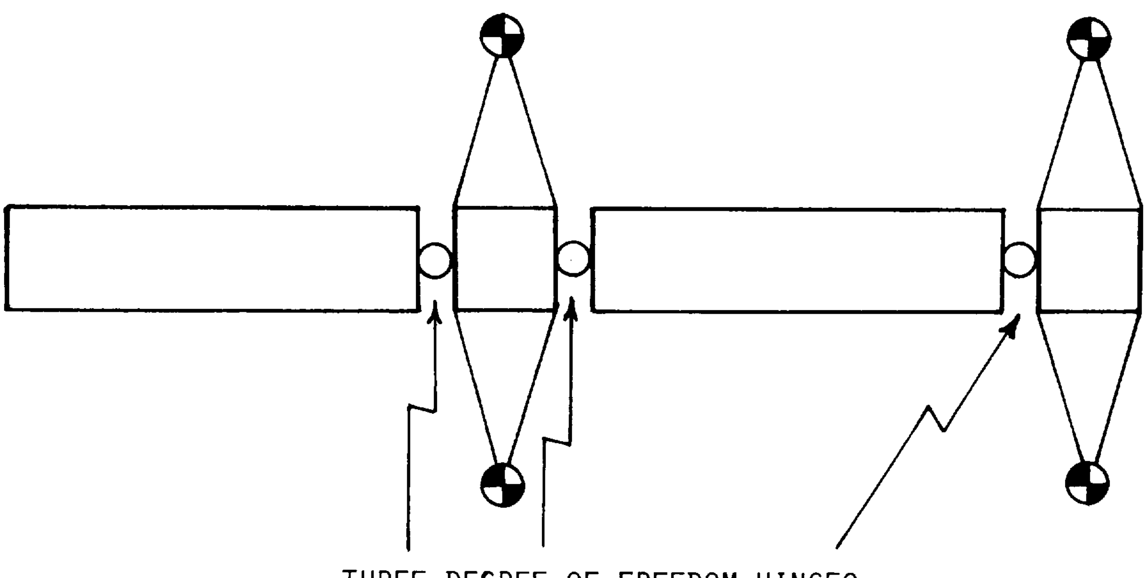
THREE AXIS FIVE BODY MODEL OF A FLEXIBLE SPACECRAFT



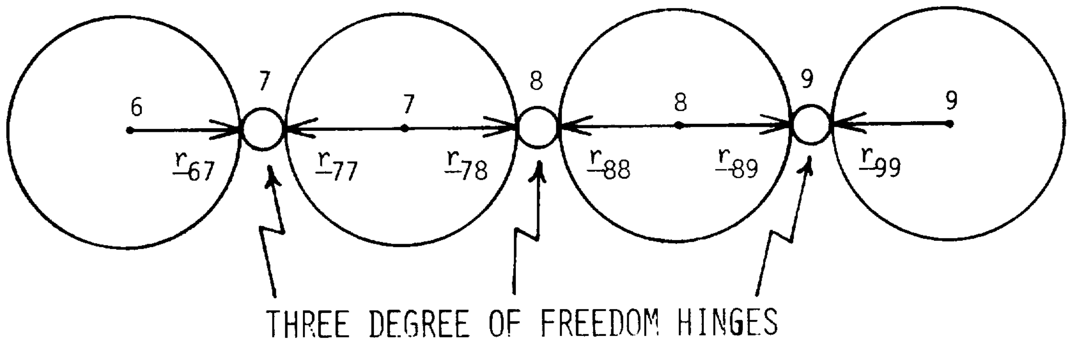
PERSPECTIVE VIEW OF HYBRID DEPLOYABLE TRUSS



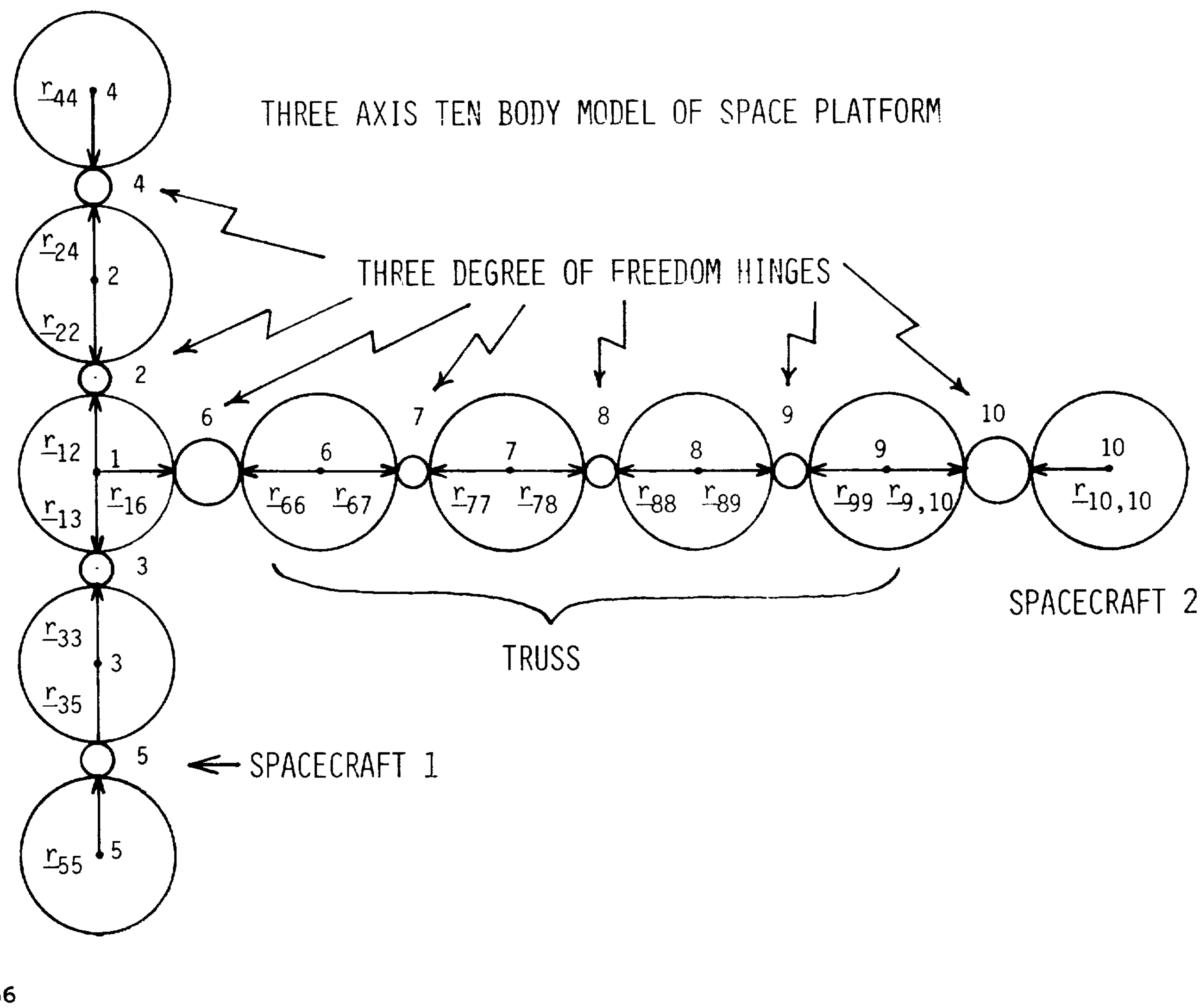
FIRST APPROXIMATION OF TRUSS



THREE AXIS FOUR BODY MODEL OF TRUSS



THREE AXIS TEN BODY MODEL OF SPACE PLATFORM



THE jTH TPBV SUBPROBLEM

$$\underline{\dot{x}}_j = A_{jj}\underline{x}_j + R_j\underline{\lambda}_j + \underline{\hat{a}}_j(t)$$

$$\underline{\dot{\lambda}}_j = -Q_j\underline{x}_j - A_{jj}^T\underline{\lambda}_j + \underline{\hat{b}}_j(t)$$

where:

$$\underline{\hat{a}}_j(t) = \sum_{\substack{k=1\\k\neq j}}^{3} (A_{jk}\underline{d}_j^k + B_{jk}\underline{s}_j^k - B_{jj}\underline{\upsilon}_j^k) \qquad \underline{\hat{b}}_j(t) = Q_j\underline{x}_{jd} - \sum_{\substack{k=1\\k\neq j}}^{3} \underline{\rho}_j^k \qquad R_j = -B_{jj}W_{ju}^{-1}B_{jj}^T$$

$\underline{x}_j(t_o) = \underline{x}_{j0}$ (initial boundary conditions)

$\underline{\lambda}_j(t_f) = 0$ (final boundary conditions)

$$\underline{\lambda}(t) = K(t)\underline{x}(t) + \underline{m}(t)$$

STATE VARIABLE ROTATIONAL DYNAMICS MODEL

$$\underline{\dot{x}}_j = \sum_{k=1}^{3} (A_{jk}\underline{x}_k + B_{jk}\underline{u}_k) \qquad j = 1, 2, 3 \qquad k = 1, 2, 3$$

where:

$$\underline{x}_k = (\underline{\omega}_k^T, \underline{\alpha}_k^T)^T$$

$$A_{jk} = \left[\begin{array}{c|c} \hat{G}_{jk}L_kC_{sk} & \hat{G}_{jk}L_kK_{sk} \\ \hline K_{jk} & [0] \end{array}\right] \qquad B_{jk} = \left[\begin{array}{c} \hat{G}_{jk} \\ \hline [0] \end{array}\right] \qquad [0] = 10 \times 10 \text{ zero matrix}$$

$$\underline{\omega}_1 = (\omega_{1x}, \omega_{2x}, \ldots, \omega_{10x})^T \qquad \underline{\omega}_2 = (\omega_{1y}, \omega_{2y}, \ldots, \omega_{10y})^T$$

$$\underline{\omega}_3 = (\omega_{1z}, \omega_{2z}, \ldots, \omega_{10z})^T$$

$\underline{u}_k$ (k = 1, 2, 3) has scalar expansion of the same form as $\underline{\omega}_k$.

$$\underline{\alpha}_1 = (\phi_1, \Delta\phi_{12}, \Delta\phi_{13}, \Delta\phi_{24}, \Delta\phi_{35}, \Delta\phi_{16}, \Delta\phi_{67}, \Delta\phi_{78}, \Delta\phi_{89}, \Delta\phi_{9,10})^T$$

$$\underline{\alpha}_2 = (\theta_1, \Delta\theta_{12}, \Delta\theta_{13}, \ldots, \Delta\theta_{9,10})^T$$

$$\underline{\alpha}_3 = (\psi_1, \Delta\psi_{12}, \Delta\psi_{13}, \ldots, \Delta\psi_{9,10})^T$$

MULTILEVEL STATE VARIABLE MODEL

$$\dot{\underline{x}}_j = A_{jj}\underline{x}_j + B_{jj}\underline{u}_j + \underline{a}_j(t) \qquad j = 1, 2, 3$$

$$\underline{a}_j = \sum_{\substack{k=1 \\ k \neq j}}^{3} (A_{jk}\underline{d}_j^k + B_{jk}\underline{s}_j^k) \qquad j = 1, 2, 3$$

$$\underline{d}_j^k = \underline{x}_k \qquad \underline{s}_j^k = \underline{u}_k \qquad k \neq j = 1, 2, 3$$

DECOMPOSED PERFORMANCE INDEX AND HAMILTONIAN

$$J = \sum_{j=1}^{3} \int_{t_o}^{t_f} P_j dt$$

where:

$$P_j = \tfrac{1}{2}(\underline{x}_j - \underline{x}_{jd})^T Q_j (\underline{x}_j - \underline{x}_{jd}) + \tfrac{1}{2}\underline{u}_j^T W_{ju} \underline{u}_j$$

$\underline{x}_{jd}$ = prespecified desired value of $\underline{x}_j$

Q_j = positive definite state variable error weighting coefficient matrix

W_{ju} = positive definite control energy weighting coefficient matrix

$$H = \sum_{j=1}^{3} H_j$$

where:

$$H_j = P_j + \underline{\lambda}_j^T \left[A_{jj}\underline{x}_j + B_{jj}\underline{u}_j + \sum_{\substack{k=1 \\ k \neq j}}^{3} (A_{jk}\underline{d}_j^k + B_{jk}\underline{s}_j^k)\right] + \sum_{\substack{k=1 \\ k \neq j}}^{3} (\underline{\rho}_j^k)^T(\underline{x}_j - \underline{d}_k^j) + (\underline{\upsilon}_j^k)^T(\underline{u}_j - \underline{s}_k^j)$$

COSTATE EQUATIONS

$$\dot{\underline{\lambda}}_j = -\frac{\partial H}{\partial \underline{x}_j} = -A_{jj}^T\underline{\lambda}_j - Q_j(\underline{x}_j - \underline{x}_{jd}) + \underline{b}_j(t)$$

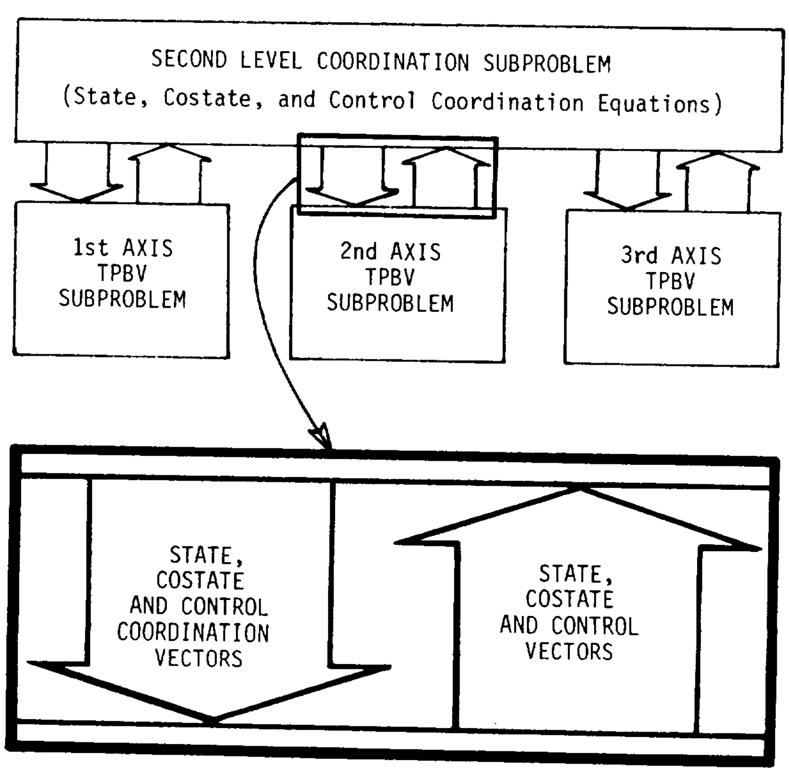
where:

$$\underline{b}_j(t) = -\sum_{\substack{k=1\\k\neq j}}^{3} \underline{\rho}_j^k \qquad\qquad \underline{\lambda}_j(t_f) = 0 \quad \text{(final boundary conditions)}$$

CONTROL EQUATIONS

$$\frac{\partial H}{\partial \underline{u}_j} = 0 \rightarrow \qquad \underline{u}_j = -W_{ju}^{-1}\Big(B_{jj}^T\underline{\lambda}_j + \sum_{\substack{k=1\\k\neq j}}^{3} \underline{\upsilon}_j^k\Big)$$

SUBPROBLEM HIERARCHY FOR HYBRID MULTILEVEL-LQR
ATTITUDE CONTROL OF THREE AXIS MODEL



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1. Chichester, F.D., "Development of a Three Axis Gauss-Seidel Multilevel Model of a Flexible Space Vehicle," Proceedings of the Twelfth Annual Pittsburgh Conference on Modeling and Simulation, April 1981, University of Pittsburgh, Pittsburgh, Pennsylvania, pp. 1303-1308.

2. Chichester, F.D., "Application of Gauss-Seidel Multilevel and LQR Control to a Three Axis Rotational Model of a Flexible Space Vehicle," Proceedings of the Twelfth Annual Pittsburgh Conference on Modeling and Simulation, April 1981, University of Pittsburgh, Pittsburgh, Pennsylvania, pp. 1309-1315.

3. Chichester, F.D. and M.T. Borelli, "A Multilevel Control Approach for a Modular Structured Space Platform," Spacecraft Guidance and Control, AGARD. (To be published.)